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THE ASSESSMENT OF MAJOR HAZARDS: USE OF THE IMPACT MODEL FOR INJURY AND DAMAGE AROUND A HAZARD SOURCE TO DETERMINE SENSITIVITY TO ERRORS IN THE CONSTITUENT MODELS

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Summary

An analytical model which describes the impact of a hazard on the surrounding area has been given previously. The basis of the model is a uniform population density, an inverse power law for the decay of the intensity of the physical effect and the lognormal distribution, or probit equation, for the relation between the causative, or injury, factor, and the probability of injury. It was shown that if these assumptions hold, the number of people injured may be approximately estimated by calculating the radius for 50% injury and assuming that all persons inside the circle suffer injury while all those outside it escape injury, that a simple correction factor can be derived to compensate for error in this method and that where the decay is inversely proportional to some power n of the distance, the correction factor is $\phi = \exp\ (2\sigma^2/n^2)$, where σ is the spread parameter of the lognormal distribution. In the present paper relationships are derived for the sensitivity of the estimate of the number of injured to errors in the parameters of this model and hence in the parameters of the constituent physical effect models and injury relations.

Introduction

An approximate method, or model, which is sometimes used to make a rapid estimate of the impact of a hazard on the surrounding area is to estimate the radius at which there is a 50% probability of injury and then to assume that all persons within this circle suffer injury while all those outside it escape injury.

It has been shown in previous papers [1,2] that a more exact model can be formulated and a correction factor derived for the error in the approximate model.

One application of this hazard impact model, which is addressed in the present paper, is the derivation of relationships for the sensitivity of the estimate of the number of injured to errors in the parameters of this model and hence in the parameters of the constituent physical effects models (fire, explosion, toxic release) and of the injury relations.

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Hazard impact model

The hazard impact model is based on the assumptions that the population density is uniform, that the intensity of the physical effect decays according to an inverse power law, that the injury factor is a power function of the intensity, and that the lognormal distribution, or probit equation, holds for the relation between the injury probability and the injury factor.

The model is given [2] by the equations

$$N_i = \pi r_{50}{}^2 d_p \phi \tag{1}$$

with

$$\phi = \exp(2\sigma^2/n^2) \tag{2}$$

where d_p is the population density (persons/m^2), N_i the total number of people injured, r_{50} the radius at which the probability P of injury is 0.5 (m), σ the spread parameter in the injury distribution and ϕ a correction factor.

The injury relation used in the model is the probit equation

$$Y = k_1 + k_2 \ln x \tag{3}$$

with

$$k_1 = 5 - m^*_n/\sigma \tag{4a}$$

$$k_2 = 1/\sigma \tag{4b}$$

where m^*_n is the normalised location parameter of the injury distribution, x the normalised injury factor and Y the probit.

In the model the intensity of the physical effect w decays with distance according to an inverse power law

$$w = k_w / r^{n_w} \tag{5}$$

where k_w is the intensity constant, n_w the intensity decay index and r the radial distance (m).

The injury factor v is a power function of the intensity

$$v = k_{vw} w^{n_{vw}} \tag{6}$$

where k_{vw} is the injury factor constant and n_{vw} the injury factor power index.

The radial distance r_{50} is given by the relation

$$r_{50} = r_o / \exp(m^*_n/n) \tag{7}$$

where n is the injury factor decay index and r_o the radius of the physical phenomenon (m). The definition of r_o is discussed in Ref. [2].

Referring to the earlier papers [1,2], it may be noted that the definition of i, the normalised intensity of the physical effect, implies the relation

$$i = w/w_o \qquad r=r \tag{8a}$$

$$= i_o = 1 \qquad r=r_o \tag{8b}$$

Likewise, the definition of x, the normalised injury factor, implies the relation

$$x = v/v_o \qquad r=r \tag{9a}$$

$$= x_o = 1 \qquad r=r_o \tag{9b}$$

An alternative unnormalised form of the probit equation is

$$Y = k_{u1} + k_{u2}\ln v \tag{10}$$

with

$$k_{u1} = 5 - m^*_u/\sigma \tag{11a}$$

$$k_{u2} = 1/\sigma \tag{11b}$$

where m^*_u is the unnormalised location parameter of the injury distribution.

The two forms of the probit equation are related through the two location parameters

$$m^*_n = m^*_u - \ln v_o \tag{12a}$$

$$v_o = \exp(m^*_u - m^*_n) \tag{12b}$$

Model sensitivity

The model may be used to derive relationships for the sensitivity of the estimate of the number of injured to errors in the parameters of the models of physical effects and of the injury relations. These sensitivity coefficients are obtained as follows.

From eqns. (5) and (6)

$$w_o = k_w/r_o^{n_w} \tag{13}$$

$$v_o = k_{vw} w_o^{n_{vw}} \tag{14}$$

Hence from eqns. (13) and (14)

$$r_o = (k_w^{1/n_w} k_{vw}^{1/n})/v_o^{1/n} \tag{15}$$

and from eqns. (15) and (12b)

$$r_o = k_w^{1/n_w} k_{vw}^{1/n} \exp[-(m^*_u - m^*_n)/n] \tag{16}$$

Hence from eqns. (16) and (7)

$$r_{50} = k_w^{1/n_w} k_{vw}^{1/n} \exp(-m^*_u/n) \tag{17}$$

Hence from eqns. (1) and (17)

$$N_i = \pi d_p k_w{}^{2/n_w} k_{vw}{}^{2/n} \exp(-2m^*{}_u/n + 2\sigma^2/n^2) \tag{18}$$

It is of interest to know the variation of N_i with respect to the following parameters:

d_p; k_w, n_w; k_{vw}, n_{vw}; $m^*{}_u, \sigma$

The normalised partial derivatives, or sensitivity coefficients, of N_i with respect to these parameters are

$$(\partial N_i/N_i)/(\partial d_p/d_p) = 1 \tag{19a}$$

$$(\partial N_i/N_i)/(\partial k_w/k_w) = 2/n_w \tag{19b}$$

$$(\partial N_i/N_i)/(\partial n_w/n_w) = -(2/n)\ln k_v + (2/n)m^*{}_u - 4\sigma^2/n^2 \tag{19c}$$

$$(\partial N_i/N_i)/(\partial k_{vw}/k_{vw}) = 2/n \tag{19d}$$

$$(\partial N_i/N_i)/(\partial n_{vw}/n_{vw}) = -(2/n)\ln k_{vw} + (2/n)m^*{}_u - 4\sigma^2/n^2 \tag{19e}$$

$$(\partial N_i/N_i)/(\partial m^*{}_u/m^*{}_u) = -2m^*{}_u/n \tag{19f}$$

$$(\partial N_i/N_i)/(\partial \sigma/\sigma) = 4\sigma^2/n^2 \tag{19g}$$

Illustrative examples

Two illustrative examples are given of the use of the model to determine the sensitivity of the estimate of the number of injured to errors in the model parameters.

Eardrum rupture by overpressure from explosion

The scenario considered is eardrum rupture by overpressure from the explosion of a condensed phase explosive TNT. The population density is 0.001 persons/m^2 (1000 persons/km^2). The mass of explosive is 1000 kg.

Then using the model of Baker et al. [3] for TNT explosions:

Mass of explosive $W = 1000$ kg

At, say

Distance $r = 50$ m

Scaled distance $z = r/W^{1/3} = 50/(1000)^{1/3} = 5$ m/kg$^{1/3}$

From curves for TNT explosions given by Baker et al. at this value of z the overpressure p° is

Overpressure $p^{\circ} = 2.8 \times 10^{4}$ N/m^2

and the slope is -1.7.

Distance r_o is chosen as 20 m.

For intensity eqn. (5) becomes

$$w = p^{\circ} = k_w/r^{n_w}$$

with

$$k_w = 2.8\times10^{4}\times(50)^{1.7} = 2.16\times10^{7}$$

$$n_w = 1.7$$

For injury factor eqn. (6) becomes

$$v = p^{\circ} = k_{vw}w^{n_{vw}}$$

with

$$k_{vw} = 1$$

$$n_{vw} = 1$$

For probability of injury use is made of the probit equation for eardrum rupture given by Eisenberg et al. [4]. The probit equations given by these authors have been summarised by Lees [5]. For eardrum rupture

$$Y = 15.6 + 1.93 \ln p^{\circ}$$

From eqns. (11a) and (11b)

$$m^*_u = 10.67$$

$$\sigma = 0.518$$

Values of the other principal variables and of the normalised sensitivity coefficients are given in Table 1.

Burn death by thermal radiation from fireball

The scenario considered is burn death by thermal radiation from a fireball of liquified propane. The population density is 0.001 persons/m^2. The mass of propane is 30 ton, the rupture is assumed to occur when the vessel contents have been heated to 60°C and the vapour pressure is 2 MPa, and the heat of combustion is 2.02×10^{6} kJ/kmol.

Then using Roberts' [6] model

Mass of propane M = 30,000 kg = 682 kmol

TABLE 1

Sensitivity coefficients for estimate of number of injured

Parameters	Example 1 (eardum rupture)	Example 2 (burn death)
d_p	0.001	0.001
k_w	2.16×10^{7}	2.67×10^{9}
n_w	1.7	2
k_{vw}	1	1.4×10^{-3}
n_{vw}	1	1.33
m^*_u	10.67	7.78
σ	0.518	0.391
r_o	20	114
k_v	2.16×10^{7}	4.82×10^{9}
n	1.7	2.67
m^*_n	−1.13	−1.87
v_o	1.33×10^{5}	1.55×10^{4}
r_{50}	38.7	230
x_{50}	0.325	0.154
N_i	5.6	173
ϕ	1.2	1.04
Sensitivity coefficients for		
d_p	1	1
k_w	1.18	1
n_w	−7.69	−11.0
k_{vw}	1.18	0.75
n_{vw}	12.2	10.7
m^*_u	−12.6	−5.83
σ	0.371	0.086

Fireball diameter (spherical symmetry) $D = 5.8(M)^{1/3}$
$= 5.8(30{,}000)^{1/3}$
$= 180$ m

Fireball diameter (hemispherical symmetry) $D = 2^{1/3}\times180$
$= 227$ m

Duration time $t_d = 0.45(M)^{1/3} = 0.45(30{,}000)^{1/3}$
$= 14$ s

Distance r_o is chosen as the fireball radius of 114 m.

Heat of combustion $(-\Delta H_c) = 2.02\times10^{6}$ kJ/kmol

Heat released $Q = 682\times2.02\times10^{6} = 1.38\times10^{9}$ kJ

Vapour pressure $P = 2$ MPa

Fraction of heat radiated $f = 0.27(P)^{0.32} = 0.27(2)^{0.32}$
$= 0.34$

Heat radiated (total) $fQ = 0.34 \times 1.38 \times 10^9 = 0.47 \times 10^9$ kJ

Heat radiated (per unit time) $E = 0.47 \times 10^9/14 = 0.336 \times 10^8$ kW

At distance r_o heat flux $I = E/4\pi r_o^2$

$$= 0.336 \times 10^8/4\pi(114)^2$$
$$= 206 \text{ kW/m}^2$$
$$= 2.06 \times 10^5 \text{ W/m}^2$$

For intensity eqn. (5) becomes

$$w = I = k_w/r^{n_w}$$

with

$$k_w = 2.06 \times 10^5 \times (114)^2 = 2.67 \times 10^9$$

$$n_w = 2$$

For injury factor use is made of the form given by Eisenberg et al. so that eqn. (6) becomes

$$v = I^{4/3} t_e/10^4 = k_{vw} w^{n_{vw}}$$

with

$$k_{vw} = 14/10^4 = 1.4 \times 10^{-3}$$

$$n_{vw} = 1.33$$

For probability of injury use is made of the probit equation for burn death given again by Eisenberg et al.

$$Y = -14.9 + 2.56 \ln(I^{4/3} t_e/10^4)$$

From eqns. (11a) and (11b)

$$m^*_u = 7.78$$
$$\sigma = 0.391$$

Values of the other principal variables and of the normalised sensitivity coefficients are given in Table 1.

Discussion

A model of hazard impact has been presented in previous papers. In the present paper the model has been used to derive relationships for the sensitivity of the estimate of the number of injured to errors in the parameters of the models for the physical effects and of the injury relations.

The model thus provides a method of analysing the relative importance of potential errors which has relevance both to the assessment of, and research on, hazards.

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List of symbols

d_p	density of population (persons/m^2)
i	normalised intensity of physical effect
I	intensity of thermal radiation (W/m^2)
k_v	second injury factor constant
k_{vw}	first injury factor constant
k_w	intensity constant
k_1, k_2	probit equation constants (normalised)
k_{u1}, k_{u2}	probit equation constants (unnormalised)
m^*	location parameter in lognormal distribution
n	decay index for injury factor
n_{vw}	power index for injury factor
n_w	decay index for intensity of physical effect
N_i	total number of people injured
p^o	peak overpressure of explosion (N/m^2)
P	probability of injury
r	radial distance (m)
r_o	radius of physical phenomenon (m)
t	time (s)
t_d	duration time (s)
t_e	exposure time (s)
v	injury factor (various units)
w	intensity of physical effect (various units)
x	normalised injury factor
σ	spread parameter in lognormal distribution (σ^2=variance)
ϕ	correction factor for variance and decay

Subscripts

n	normalised
o	at r_o
u	unnormalised
50	for probability of injury equal to 0.5

Other variables used in the illustrative examples are defined locally.

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